Physics of $\sin^2 2\theta_{13}$

★ **What is θ_{13} ?**

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{e1} & V_{e2} & V_{e3} \\ V_{\mu 1} & V_{\mu 2} & V_{\mu 3} \\ V_{\tau 1} & V_{\tau 2} & V_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad (\text{with } |V_{e3}| \equiv \sin\theta_{13})$$

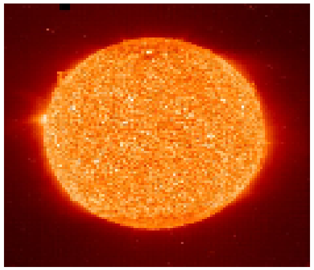
θ_{13} **is the yet unknown (smallest) lepton mixing angle.**

★ **What does $\sin^2 2\theta_{13}$ mean?**

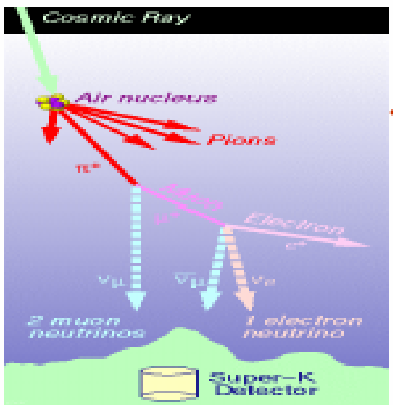
$$P(\overline{\nu_e} \to \overline{\nu_e}) \approx 1 - \sin^2 2\theta_{13} \cdot \sin^2\left(1.27\frac{\Delta m^2_{atm} L}{E}\right) \qquad (\text{Reactor})$$

$\sin^2 2\theta_{13}$ **measures the oscillation amplitude of reactor neutrinos, e.g., at Daya Bay (大亚湾）.**

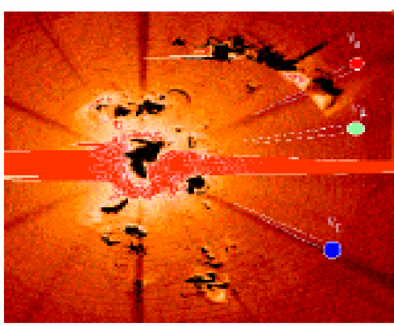
Neutrino Sources and Topics:



←Sun

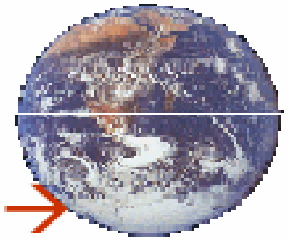


←Atmosphere



←Big Bang

Earth→



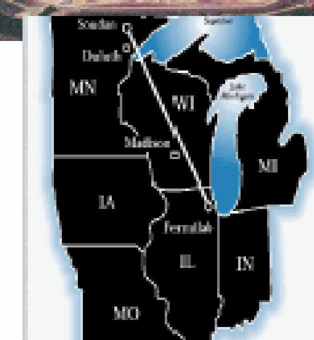
Supernovae→

Astronomy

Reactors→

Accelerators→

Laboratories



Physics of $\sin^2 2\theta_{13}$

Zhi-zhong Xing/IHEP (邢志忠/高能所)

OUTLINE

★ High Impact of θ_{13} on Particle Physics

★ $\theta_{13} = 0$: Prerequisite and Consequences

★ $\theta_{13} \neq 0$: How Big or How Small?

☆ Expectations from Global Analyses

☆ Expectations from Specific Models

★ Comments and Concluding Remarks

High Impact of θ_{13} on Particle Physics

Strong experimental evidence in favor of neutrino oscillations (SK, SNO, KamLAND, K2K, …) implies that

- ★ **neutrinos are massive**
- ★ **lepton flavors are mixed**

A major breakthrough in today's particle physics

In SM, lepton number is conserving & neutrinos are massless Weyl particles. But many people did not believe that, because

- ☆ $m_\nu = 0$ **is not guaranteed by fundamental law or symmetry**
- ☆ $m_\nu \neq 0$ **is naturally expected in grand unified theories**

Today we are convinced that $m_\nu \neq 0$ is actually true. Then,

- ☆ **SM incomplete—an experimental reason to go beyond SM**
- ☆ **lepton mixing at the front of experimental particle physics**

Quark flavor mixing (CKM matrix):

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & e^{-i\delta} & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Experimental steps: $\theta_{12} \rightarrow \theta_{23} \rightarrow \theta_{13} \rightarrow \delta$

$\sim 13^\circ$ $\sim 2^\circ$ $\sim 0.2^\circ$ $\sim 65^\circ$

Lepton flavor mixing (MNS matrix):

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & e^{-i\delta} & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Experimental steps: $\theta_{23} \rightarrow \theta_{12} \rightarrow \theta_{13} \rightarrow \delta / \rho / \sigma$

$\sim 45^\circ$ $\sim 33^\circ$ $< 13^\circ$ $\sim ???$

Lessons of quark mixing for lepton mixing:

★ **The smallest mixing angle θ_{13} is a crucial turning-point in doing precision measurements and detecting CP violation**

★ **Neutrino masses might have a relatively weak hierarchy (or near degeneracy) relevant to the bi-large mixing pattern**

Why is the measurement of θ_{13} extremely important?

★ θ_{13} is a fundamental parameter of flavor physics

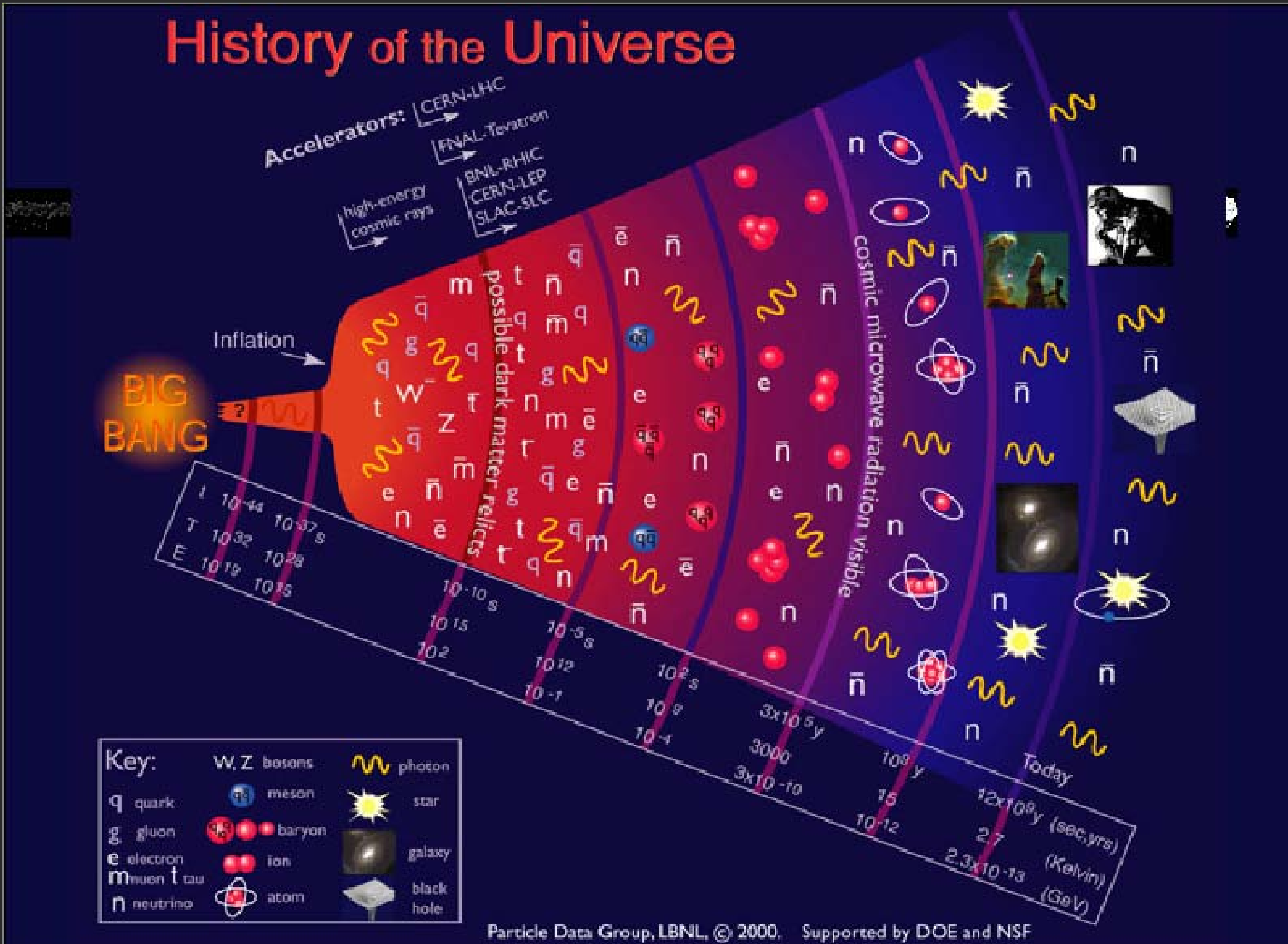
★ θ_{13} controls the observable effects of CP violation

★ θ_{13} is a sensitive model / theory discriminator

★ θ_{13} is a key to search for new and newer physics

Possible impact of θ_{13} on astrophysics and cosmology:

Dark matter, Matter-Antimatter asymmetry, Supernovae, ...


History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10^{-44} 10^{-37} s
T 10^{32} 10^{28}
E 10^{19} 10^{15}
10^{-10} s
10^{15}
10^{2}
10^{-5} s
10^{12}
10^{-1}
10^{2} s
10^{9}
10^{-4}
3×10^{5} y
3000
3×10^{-10}
10^{9} y
15
10^{-12}
Today
12×10^{9} y (sec,yrs)
2.7 (Kelvin)
2.3×10^{-13} (GeV)
Key:
q quark
g gluon
e electron
m muon t tau
n neutrino
W, Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

$\theta_{13} = 0$: Prerequisite and Consequences

Experimentally, $\theta_{13} < 13^\circ$ (CHOOZ), $\theta_{13} = 0$ is not impossible
Theoretically, there is no good reason (symmetry) for $\theta_{13} = 0$

Note that $\theta_{13} = 0$ would hold, if there existed mass degeneracy of charged leptons or neutrinos (with Majorana phases Ψ_i):

☆ $m_1 = m_3$ **with** $\Psi_1 = \Psi_3$ **or** $m_2 = m_3$ **with** $\Psi_2 = \Psi_3$

☆ $m_e = m_\mu$ **or** $m_e = m_\tau$ **(J.W. Mei, Z.Z.X., 2003)**

But all these conceptually interesting limits are unrealistic!

Consequences of $\theta_{13} = 0$: ▲ One CP-violating phase would vanish; ▲ Leptonic unitarity triangles would collapse — CP violation would not appear in neutrino oscillations; ▲ There would be no matter effects on m_3 and θ_{23}; etc.

$\theta_{13} \neq 0$: How Big or How Small?

Convincing flavor theory has been lacking—it is at present impossible to predict fermion masses, flavor mixing angles and CP phases at a fundamental level—the flavor problem

Flavor experiments play the leading role — determine those unknown parameters and shed light on the unknown theory

★ <u>Current experimental data</u> allow us to get useful hints on the size of θ_{13} from global analyses of solar, atmospheric, reactor and accelerator neutrino oscillation data (like good lesson from pinning down m_t with LEP electroweak data and $\sin 2\beta$ with non-B factory quark mixing data) — essentially model-independent.

★ <u>Current experimental data</u> help us to build some realistic models from which the size of θ_{13} can be calculated.

Expectations from Global Analyses

Group A: J. Bahcall, M. Gonzalez-Garcia, C. Pena-Garay (**BGP**)

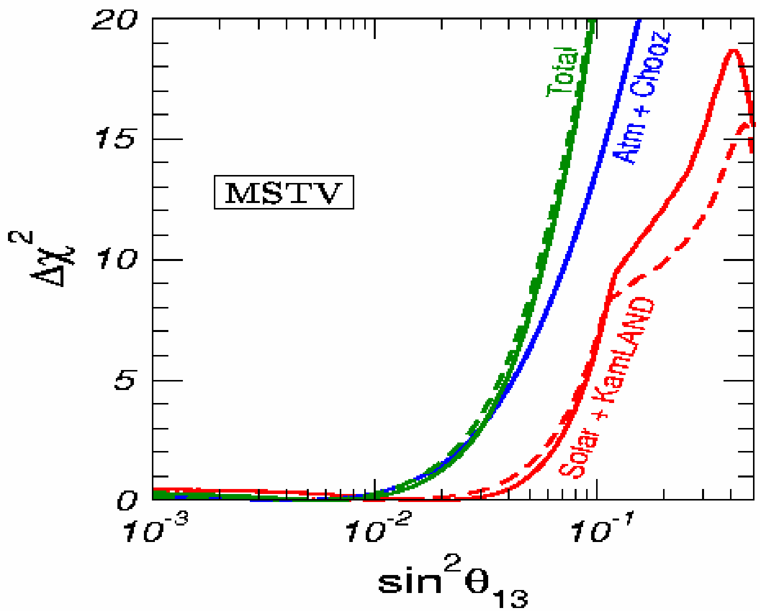
Group B: M. Maltoni, T. Schwetz, M. Tortola, J. Valle (**MSTV**)

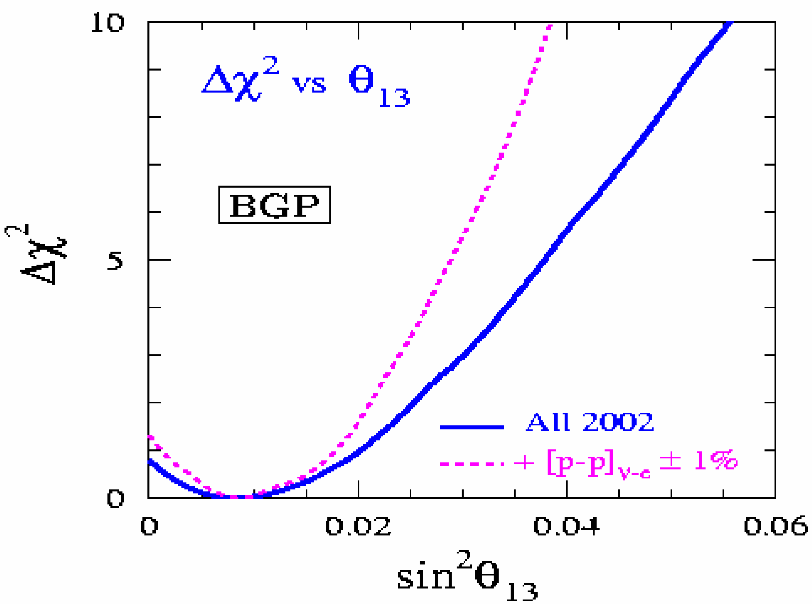
Main approximation: one mass scale dominance for 3 types of experimental data ($\Delta m^2_{sun} \ll \Delta m^2_{atm}$, δ is decoupled)

☆ **Solar and KamLAND**: Δm^2_{21} , θ_{12}, θ_{13} ;

☆ **Atmospheric and K2K**: Δm^2_{32} , θ_{23}, θ_{13} ;

☆ **CHOOZ**: Δm^2_{32} , θ_{13} .





★ **Best fit of BGP:** $\theta_{13} = 5.44^\circ$ **or** $\sin^2 2\theta_{13} = 0.036$

★ **Best fit of MSTV:** $\theta_{13} = 4.44^\circ$ **or** $\sin^2 2\theta_{13} = 0.024$

An experiment with sensitivity 1% may measure $\sin^2 2\theta_{13}$!

Parameter (BGP [2])	Best fit	95% C.L.	3σ interval
Δm^2_{21} (10^{-5} eV^2)	7.1	5.8–8.9	4.6–10.8
Δm^2_{32} (10^{-3} eV^2)	2.6	1.6–3.6	1.4–4.1
$\tan^2\theta_{12}$	0.45	0.31–0.73	0.27–0.98
$\tan^2\theta_{23}$	1.0	0.52–2.19	0.44–2.73
$\sin^2\theta_{13}$	0.009	≤ 0.042	≤ 0.061

Parameter (MSTV [3])	Best fit	2σ interval	3σ interval
Δm^2_{21} (10^{-5} eV^2)	6.9	6.0–8.4	5.4–9.5
Δm^2_{31} (10^{-3} eV^2)	2.6	1.8–3.3	1.4–3.7
$\sin^2\theta_{12}$	0.30	0.25–0.36	0.23–0.39
$\sin^2\theta_{23}$	0.52	0.36–0.67	0.31–0.72
$\sin^2\theta_{13}$	0.006	≤ 0.035	≤ 0.054

Expectations from Specific Models

Low–energy phenomenology of lepton masses and mixing:
M_l –charged lepton mass matrix ; M_ν –effective neutrino mass matrix

$$U_l^\dagger M_l U_l' = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad U_\nu^\dagger M_\nu U_\nu^* = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad \boxed{V = U_l^\dagger U_\nu}$$

Totally 12 parameters, but only 7 of them have been known

M_l and M_ν may stem from GUTs or non–GUT models, but their structures are in general unspecified. Testable relation between lepton masses and flavor mixing parameters can't be achieved, unless phenomenological hypotheses are made.

Results for θ_{13} from "predictive" models are of two types:

- ★ θ_{13} **is given in terms of mass ratios of leptons or quarks**
- ★ θ_{13} **is given by other known ν — oscillation parameters**

“Democratic” neutrino mixing model (H. Fritzsch, Z.Z. X., 1996)

Idea: $S(3)_L \times S(3)_R$ symmetry of M_l and $S(3)$ symmetry of M_ν are explicitly broken by small perturbations

$$M_l = \frac{c_l}{3}\left[\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix} + \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \varepsilon_l\end{pmatrix} + \begin{pmatrix}-i\delta_l & 0 & 0\\ 0 & i\delta_l & 0\\ 0 & 0 & 0\end{pmatrix}\right],$$

$$M_\nu = c_\nu\left[\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} + \begin{pmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \varepsilon_\nu\end{pmatrix} + \begin{pmatrix}-\delta_\nu & 0 & 0\\ 0 & \delta_\nu & 0\\ 0 & 0 & 0\end{pmatrix}\right].$$

Bi-large lepton flavor mixing with $\theta_{13} \neq 0$ and CP violation:

$$V = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{pmatrix} + i\sqrt{\frac{m_e}{m_\mu}}\begin{pmatrix}\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}}\\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0\\ 0 & 0 & 0\end{pmatrix} + \frac{m_\mu}{m_\tau}\begin{pmatrix}0 & 0 & 0\\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}}\\ \frac{-1}{\sqrt{12}} & \frac{-1}{\sqrt{12}} & \frac{1}{\sqrt{3}}\end{pmatrix}$$

Prediction $\sin\theta_{13} \approx 2/\sqrt{6} \times \sqrt{m_e/m_\mu} \approx 0.057$ Or, $\sin^2 2\theta_{13} = 0.013$

SO(10)–inspired neutrino model (W. Buchmüller, D. Wyler, 2001)

Idea: **Lepton-quark symmetry** $\epsilon_u \sim 0.07$ and $\epsilon_d \sim 0.21$ **from data**

$$M_D \sim M_u \sim m_t \begin{pmatrix} 0 & \mathcal{O}(\epsilon_u^3) & 0 \\ \mathcal{O}(\epsilon_u^3) & \mathcal{O}(\epsilon_u^2) & \mathcal{O}(\epsilon_u^2) \\ 0 & \mathcal{O}(\epsilon_u^2) & \mathcal{O}(1) \end{pmatrix}, \quad M_l \sim M_d \sim m_b \begin{pmatrix} 0 & \mathcal{O}(\epsilon_d^3) & 0 \\ \mathcal{O}(\epsilon_d^3) & \mathcal{O}(\epsilon_d^2) & \mathcal{O}(\epsilon_d^2) \\ 0 & \mathcal{O}(\epsilon_d^2) & \mathcal{O}(1) \end{pmatrix}$$

Seesaw mechanism $M_\nu \approx M_D M_R^{-1} M_D^T$ **to obtain light neutrinos**

$$M_R \sim M_3 \begin{pmatrix} 0 & \mathcal{O}(\epsilon_u^5) & 0 \\ \mathcal{O}(\epsilon_u^5) & \mathcal{O}(\epsilon_u^4) & \mathcal{O}(\epsilon_u^4) \\ 0 & \mathcal{O}(\epsilon_u^4) & \mathcal{O}(1) \end{pmatrix}, \quad M_\nu \sim \frac{m_t^2}{M_3} \begin{pmatrix} 0 & \mathcal{O}(\epsilon_u) & 0 \\ \mathcal{O}(\epsilon_u) & \mathcal{O}(1) & \mathcal{O}(1) \\ 0 & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

Mass spectra $m_1 : m_2 : m_3 \sim \epsilon_u : \epsilon_u : 1$ **and** $M_1 : M_2 : M_3 \sim \epsilon_u^6 : \epsilon_u^4 : 1$

It is possible to get bi-large mixing pattern with a prediction

$\sin\theta_{13} \sim 1/\sqrt{2} \times \sin\theta_C \approx 0.155$ **i.e. $\sin^2 2\theta_{13} = 0.094$. Too good?**

Comments: leptogenesis can be accommodated; RGE effects are not included; difficult to get a satisfactory flavor picture

Texture zeros of M_ν (P. Frampton, S. Glashow, D. Marfatia, 2002)

Pattern	Texture of M_ν	Prediction for θ_{13}
A_1	$\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$	$\sin\theta_z \approx \sqrt{\dfrac{R_\nu \tan^2\theta_x}{\tan^2\theta_y \lvert 1-\tan^4\theta_x \rvert}}$
A_2	$\begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}$	$\sin\theta_z \approx \sqrt{\dfrac{R_\nu \tan^2\theta_x \tan^2\theta_y}{\lvert 1-\tan^4\theta_x \rvert}}$
B_1	$\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$	$\sin\theta_z \approx \dfrac{R_\nu \tan\theta_x}{(1+\tan^2\theta_x)\lvert \tan 2\theta_y \cos\delta \rvert}$
B_2	$\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$	$\sin\theta_z \approx \dfrac{R_\nu \tan\theta_x}{(1+\tan^2\theta_x)\lvert \tan 2\theta_y \cos\delta \rvert}$
B_3	$\begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$	$\sin\theta_z \approx \dfrac{R_\nu \tan\theta_x}{(1+\tan^2\theta_x)\tan^2\theta_y \lvert \tan 2\theta_y \cos\delta \rvert}$
B_4	$\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}$	$\sin\theta_z \approx \dfrac{R_\nu \tan\theta_x \tan^2\theta_y}{(1+\tan^2\theta_x)\lvert \tan 2\theta_y \cos\delta \rvert}$
C	$\begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$	$\sin\theta_z \sim \dfrac{1}{\tan 2\theta_x \tan 2\theta_y \cos\delta}$

$R_\nu \equiv \Delta m^2_{sun} / \Delta m^2_{atm}$. We get $\theta_{13} \sim 5^\circ$ or $\sin^2 2\theta_{13} = 0.03$

Minimal seesaw ν model (P. Frampton, S. Glashow, T. Yanagida, 2002)

Motivation: to simultaneously interpret neutrino oscillations and cosmological baryon number asymmetry

$$-\mathcal{L}_{\rm mass} = \overline{(\nu_e, \nu_\mu, \nu_\tau)}\, M_D \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} + \frac{1}{2}\overline{(N_1^c, N_2^c)}\, M_R \begin{pmatrix} N_1 \\ N_2 \end{pmatrix} + \text{h.c.}$$

In the basis of diagonal M_l and M_R, FGY ansatz of M_D gives

$$M_D = \begin{pmatrix} a & 0 \\ a' & b \\ 0 & b' \end{pmatrix}, \quad M_\nu \approx M_D M_R^{-1} M_D^T = \begin{pmatrix} \frac{a^2}{M_1} & \frac{aa'}{M_1} & 0 \\ \frac{aa'}{M_1} & \frac{(a')^2}{M_1} + \frac{b^2}{M_2} & \frac{bb'}{M_2} \\ 0 & \frac{bb'}{M_2} & \frac{(b')^2}{M_2} \end{pmatrix}$$

Since $|\mathrm{Det}(M_\nu)| = m_1 m_2 m_3 = 0$, $m_1 = 0$ or $m_3 = 0$ must hold.

★ The texture of M_ν is stable against radiative corrections;

★ CP-violating phases of M_ν can be determined by θ_{13} etc.

We take the $m_1 = 0$ case (normal hierarchy) as an example:

Two CP-violating phases (W.L. Guo / J.W. Mei, Z.Z. X., 2003):

$$\delta = \arccos\left[\frac{c_y^2 s_z^2 - R_\nu s_x^2 \left(c_x^2 s_y^2 + s_x^2 c_y^2 s_z^2\right)}{2 R_\nu s_x^3 c_x s_y c_y s_z}\right]$$

$$\sigma = \frac{1}{2}\arctan\left[\frac{c_x s_y \sin\delta}{s_x c_y s_z + c_x s_y \cos\delta}\right]$$

$|\cos\delta| < 1$ requires $\sin\theta_{13} \sim 0.075$, in a very restrictive range

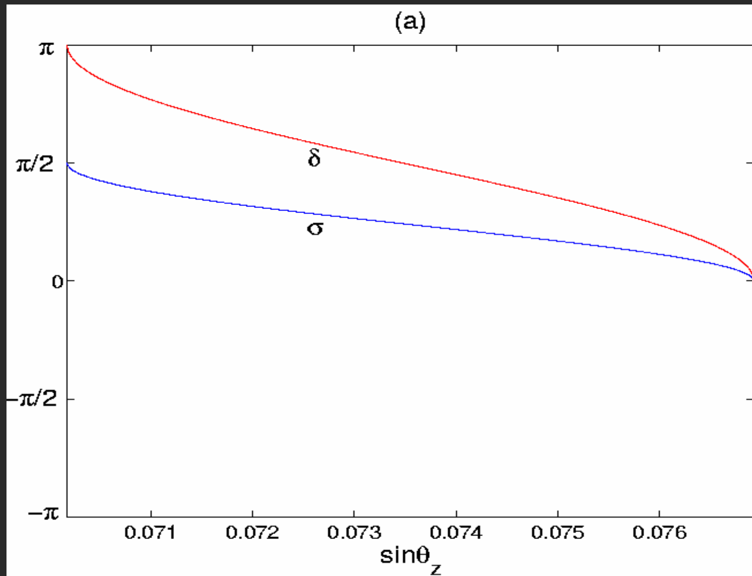
Cosmological baryon number asymmetry via leptogenesis:

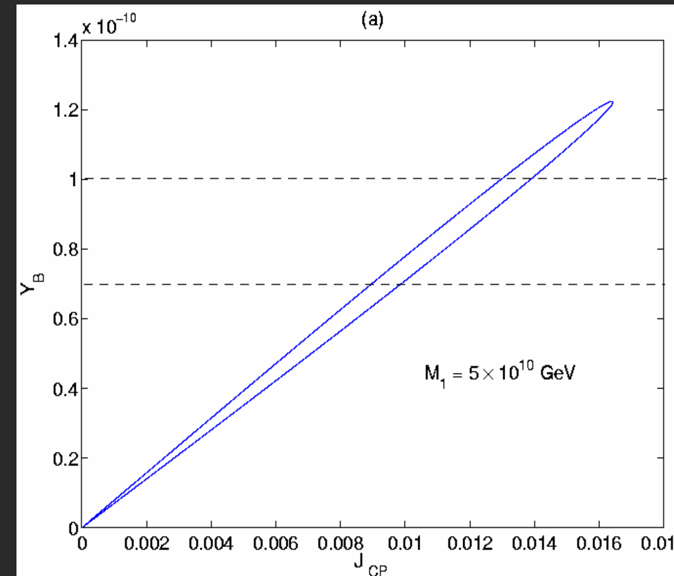
$$\varepsilon_1 \equiv \frac{\Gamma(N_1 \to l + H) - \Gamma(N_1 \to \bar{l} + H^*)}{\Gamma(N_1 \to l + H) + \Gamma(N_1 \to \bar{l} + H^*)}$$

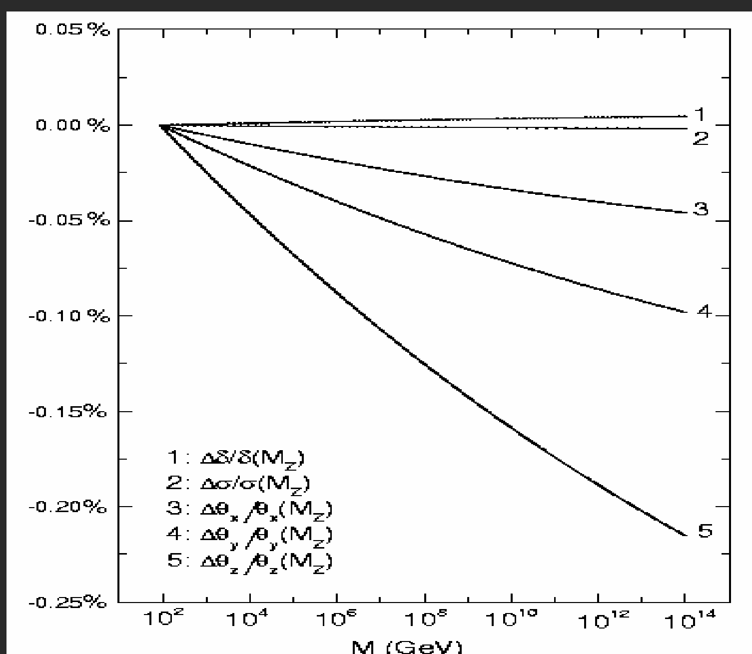
$$\propto M_1 \cdot \sin\left[\arctan\left(\frac{\sqrt{R_\nu}\; s_x^2 c_z^2 \sin 2\sigma}{s_z^2 + \sqrt{R_\nu}\; s_x^2 c_z^2 \cos 2\sigma}\right)\right]$$

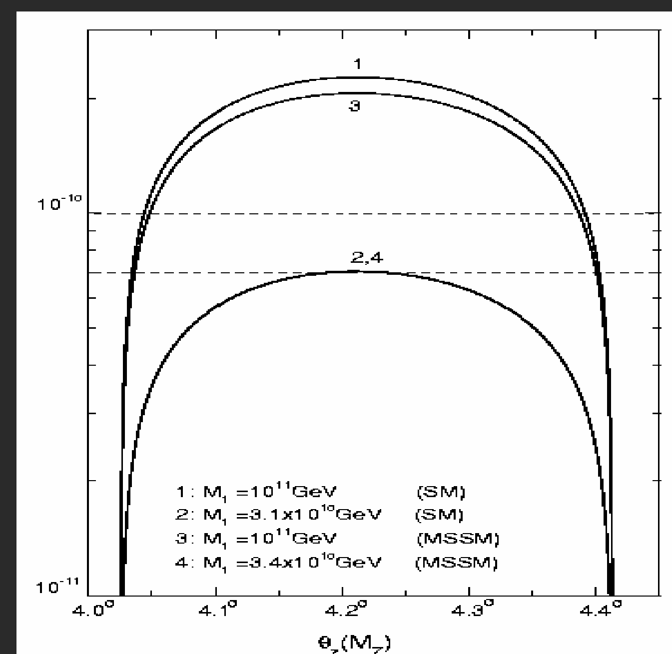
$Y_B \propto \varepsilon_1 \sim 10^{-10}$ can be achieved, if $M_1 \geqslant 10^{10}$ GeV (SM/SUSY)

Remark: a testable model in which θ_{13} determines Y_B etc.


(a)
π
π/2
0
−π/2
−π
δ
σ
0.071
0.072
0.073
0.074
0.075
0.076
sinθz


(a)
x 10^−10
1.4
1.2
1
0.8
0.6
0.4
0.2
0
YB
M1 = 5×10^10 GeV
0
0.002
0.004
0.006
0.008
0.01
0.012
0.014
0.016
0.018
J CP


0.05%
0.00%
-0.05%
-0.10%
-0.15%
-0.20%
-0.25%
1
2
3
4
5
1: Δδ/δ(MZ)
2: Δσ/σ(MZ)
3: Δθx/θx(MZ)
4: Δθy/θy(MZ)
5: Δθz/θz(MZ)
10^2
10^4
10^6
10^8
10^10
10^12
10^14
M1(GeV)


1
3
2,4
10^−10
10^−11
1: M1 =10^11GeV (SM)
2: M1 =3.1x10^10GeV (SM)
3: M1 =10^11GeV (MSSM)
4: M1 =3.4x10^10GeV (MSSM)
4.0°
4.1°
4.2°
4.3°
4.4°
θz(MZ)

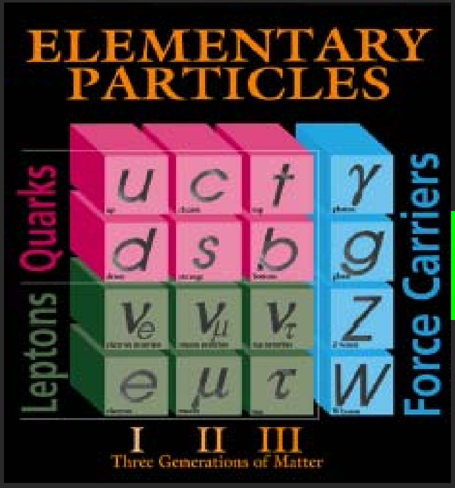
Comments and Concluding Remarks

★ **Another bonus** to be achieved from a measurement of θ_{13} is to understand **sub-leading** effects in neutrino oscillations & to probe or isolate much **newer** physics (e.g., **sterile** neutrinos)

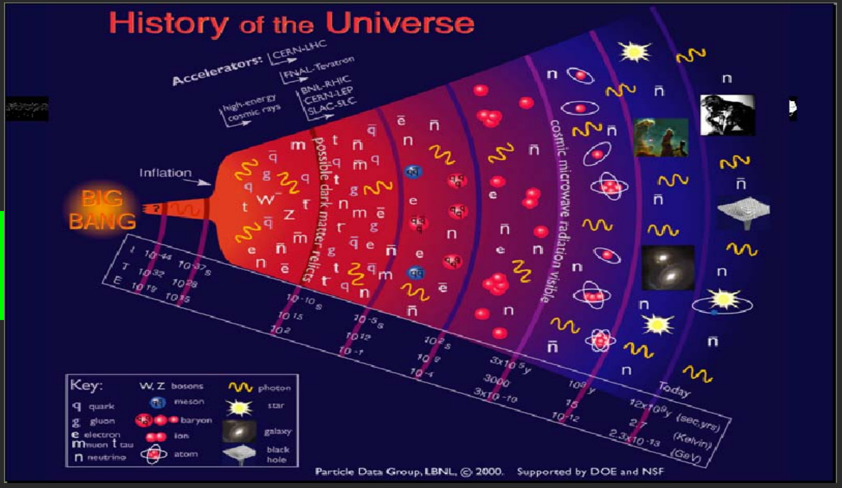
★ More theoretical reasons for $\sin^2 2\theta_{13} > 0.01$ (M. Lindner 2003)

☆ Models with the "**anarchy**" idea typically predict $\sin^2 2\theta_{13}$ just below its current upper limit;

☆ Quantum corrections allow $\sin^2 2\theta_{13} = 0$ at the **GUT** scale to become $\sin^2 2\theta_{13} > 0.01$ at low scales: **nothing** → **something**.



θ_{13} **has a role!**



Thank You

&

Happy Chinese New Year